Lateral Pressure and Bedding Factors

INTRODUCTION

The application of lateral pressure acting on a buried pipe has been subject to much discussion over at least the past quarter century, particularly in determining the relationship of pipe strength in a three-edge bearing (T.E.B.) test to the structural strength of the pipe in the field. This relationship, called the bedding factor (B_f), allows the designer to determine the required field supporting strength of the pipe for a given installation.

Previously published data addressing bedding factors are based on experiments conducted in the early 1900's by Marston, Schlick and Spangler. Bedding factors for trench conditions were developed from nine test installations which included three each of concrete, clay and cast iron pipe. These installations were configured with no soil, and consequently no lateral pressure, on the sides of the pipe. The trench condition bedding factors were originally presented as a single value for each bedding class. Design Data 38, "Bedding Factors-Trench Installations," is an initial

Current pipeline backfill installation equipment and methods, (below) have changed dramatically since the 1920's (above).

methodology for determining appropriate bedding factors based on the assumption that lateral pressure is effective in the trench condition and becomes more significant as trench width increases. From later research on the embankment condition by Marston et. al., an equation was developed for embankment bedding factors which partially recognizes the effects of active lateral pressure.

Although compaction technology and soil density requirements for installation of pipe systems have changed significantly, the early developed bedding factors generally are still being used. Current bedding factors typically used for concrete pipe are quite conservative and significantly understate the real supporting strength of concrete pipe. In addition, installations are generally specified with select or granular backfill material compacted to a density of 85-95% surrounding the pipe. Both the equipment providing the compactive effort, and the equipment used to verify that such compaction has been achieved, have improved dramatically since the research of the 1920's. This Buried Fact presents a method of determining bedding factors which consider lateral forces and reflect current installation methods.

Rankine's coefficient of active earth pressure (K_a) varies from 0.33 to 0.37 for commonly encountered soils. A value of $K_a = 0.33$ has long been established as a conservative estimate for most situations. Since the coefficient of passive earth pressure (K_p) is the inverse of the active earth pressure coefficient, values of up to 3.0 are not uncommon. For this reason, passive lateral earth pressures can be much higher than the active earth pressures.

MAXIMUM MOMENTS

To calculate the appropriate bedding factors to be used for design, determination of the maximum moments developed for each loading condition are necessary. Due to the weight of the pipe, the maximum moment will occur at the pipe invert. This Buried Fact utilizes the paper by James M. Paris, "Stress Coefficients for Large Horizontal Pipes" (Engineering News Record, Vol. 87, No. 19, November, 1912), which presents coefficients for determining bending moments for various combinations of loadings and beddings. Since the pipe weight is present in all load examples, this load will be omitted from the comparative calculations. The equations for the maximum external load moment at the invert for representative cases are presented in the following paragraphs.

Three-edge Bearing Test

The maximum moment developed in a pipe subject to a three-edge bearing test (*Figure 2*) where the applied load is Q, and the mean pipe diameter is D_m, can be determined from Paris' Case I (*Figure 3*). Assume the bearing strips to be sufficiently close so as to act as a point support and neglect the weight of the pipe. A point load is applied during the T.E.B. test at the crown (point T), therefore $\theta = 0$ and $Q = 2P$. Paris' maximum moment at the invert (Point B) is:

$$M_{TEB} = +0.637\,Pr_m \tag{6}$$

where:

$$r_m = \frac{D_m}{2}, P = \frac{Q}{2} \tag{7}$$

and:

$Q =$ T.E.B. load
$D_m =$ mean pipe diameter
$r_m =$ mean pipe radius

therefore:

$$M_{TEB} = +0.637 \left(\frac{Q}{2}\right)\left(\frac{D_m}{2}\right) \tag{8}$$

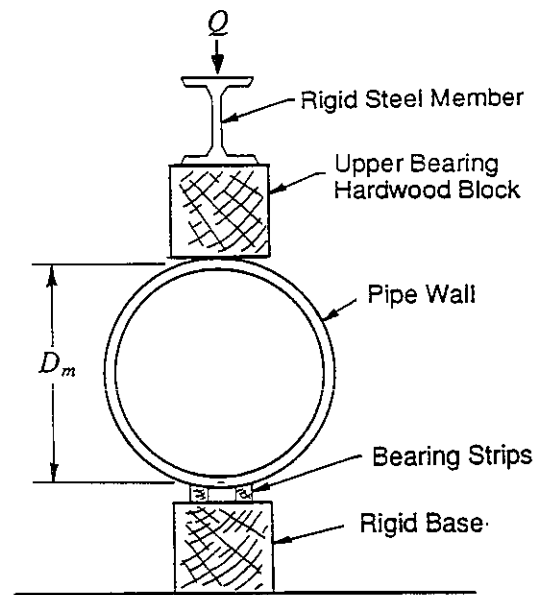and:

$$M_{TEB} = +0.159\,QD_m \tag{9}$$
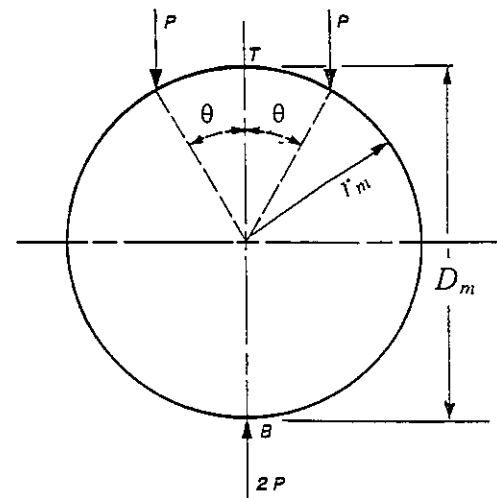

Figure 2. Three-Edge Bearing Test.

Figure 3. Paris' Case 1, Three-Edge Bearing Test.

Buried Pipe - No Lateral Pressure

Determine the maximum external load moment in the field for a pipe installed with a Class B bedding, but with voids on each side of the pipe such that lateral forces do not exist. In other words, assume the backfill at the sides of the pipe does not contact the pipe. This is an improbable condition but is equivalent to Spangler's early work.

The pipe experiences two loads under these conditions; one, a uniform earth load on top of the pipe and, two, a supporting earth pressure at the bottom of the pipe.

BEDDING FACTORS

Bedding factors relate three-edge bearing test strengths to required field strengths and can be derived by equating the calculated moment developed in three-edge bearing to the calculated moment developed in the buried condition using the soil prism loads. The ratio of these moments is the bedding factor.

$$B_f = \frac{M_{TEB}}{M_{FIELD}} \qquad (1)$$

The maximum moment in an installed rigid pipe, and therefore the bedding factor, depends on two characteristics of the installation:

☐ width of the bedding of the pipe,
☐ magnitude of the lateral pressure against the sides of the pipe.

BEDDING WIDTH

The width of the bedding of a circular pipe is defined by a bedding angle, a horizontal dimension or a vertical dimension (see *Figure 1*) over which the vertical reactive force on the bottom of the pipe is distributed. In his publications, Spangler presented central bedding angles of 90°, 60°and a line loading for the classic B, C and D beddings respectively. However, he also provided bedding widths which did not always correlate with the angles. *Table 1* presents currently accepted values for bedding dimensions as normally constructed. In addition, bedding dimensions are given for a true 90° bedding condition.
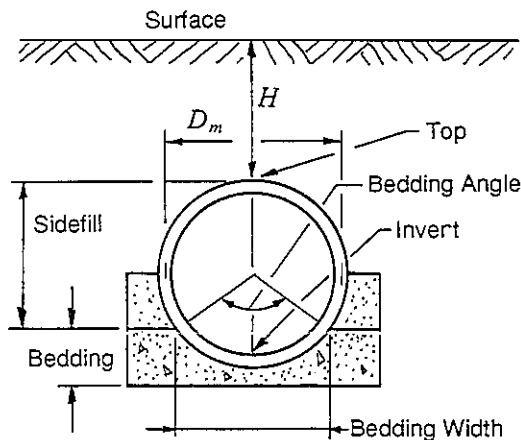

Figure 1. Bedding Dimensions.

LATERAL PRESSURE

One of the characteristics of soil is its ability to exert a lateral pressure against objects with which it comes in contact. This lateral pressure can be an active pressure in which the soil exerts a force against the object by virtue of the soil's tendency to slip laterally and seek its natural angle of repose. Lateral pressure can also be of a passive nature in which the object moves towards the soil, developing a resistive pressure.

Backfilling practices and pipe deflection characteristics influence the development and magnitude of lateral pressures on the sides of the pipe. The presence of lateral pressure causes bending moments in the opposite direction from those produced by vertical loads and bedding support reactions, therefore decreasing the magnitude of bending moments in the pipe wall. This reduction in bending moments results in an equivalent increase in the pipe's supporting strength.

The lateral pressure on a buried circular conduit with well compacted backfill varies depending upon the deflection of the conduit. With no deflection occurring in the conduit, the maximum lateral pressure will be an active earth pressure. In cases where pipe deflection occurs, passive lateral pressures are generated as the pipe deforms slightly under vertical load, increasing its horizontal dimension and pushing against the soil. This is particularly true when high soil compaction levels are used in backfilling around the pipe.

RANKINE'S RATIO

In the late 1800's Rankine developed one of the classical earth pressure theories relating the magnitude of lateral pressure to vertical pressure. Rankine described the active earth pressure generated in a soil mass of dry cohesionless material, typical of pipe backfill, by the following equation:

$$P_a = K_a \times w \qquad (2)$$

where:

P_a = lateral active earth pressure

w = vertical pressure on any horizontal plane in backfill

K_a = Rankine's active earth pressure coefficient

and where:

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \qquad (3)$$

where:

ϕ = angle of internal friction of the soil

Passive earth pressures, generated when the pipe deflects and pushes against the soil, are described by the following equation:

$$P_p = K_p \times w \qquad (4)$$

where:

P_p = lateral passive earth pressure

K_p = Rankine's passive earth pressure coefficient

and where:

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \qquad (5)$$

The uniform earth load acts across the full width of the top of the pipe, equivalent to Paris' Case 7 (*Figure 4*) where:

w_{SOIL} = vertical earth load at the top of the pipe, pounds per square foot.

and the maximum moment at the invert is:

$$M_{SOIL} = +0.587 w_{SOIL} r_m^2 \tag{10}$$

since:

$$r_m = \frac{D_m}{2}$$

and:

$$W_E = w_{SOIL} D_m \tag{11}$$

where:

W_E = total uniform earth load on top of the pipe, pounds per linear foot.

The maximum moment at the invert due to the soil load is:

$$M_{SOIL} = +0.147 W_E D_m \tag{12}$$

The supporting earth pressure at the bottom of the pipe is distributed over the central bedding angle. The resulting moment at the pipe invert (from Paris' Case 8, *Figure 5*), assuming a bedding angle (2α) of 75° is:

$$\phi_B = 37.5°$$
$$M_{BED} = -0.038 w_{BED} D_m^2 \tag{13}$$

And the reaction at the base is:

$$R_{BED} = 2(\sin \alpha)(w_{BED})\left(\frac{D_m}{2}\right) \tag{14}$$

$$R_{BED} = +0.609 w_{BED} D_m \tag{15}$$

Because the system is in equilibrium, the earth load on top of the pipe results in an equal and opposite supporting earth pressure at the base of the pipe. Setting these forces equal to each other:

$$W_E = R_{BED} \tag{16}$$

$$W_E = +0.609 w_{BED} D_m \tag{17}$$

then:

$$w_{BED} = 1.64 \frac{W_E}{D_m} \tag{18}$$

Using the principal of superposition, the maximum moment at the invert, for a pipe in the field with no lateral pressure, is the combination of moments due to the uniform earth load on top of the pipe (Equation 12)
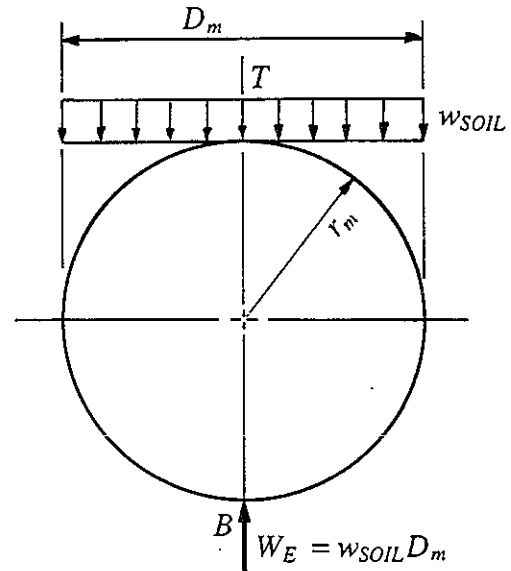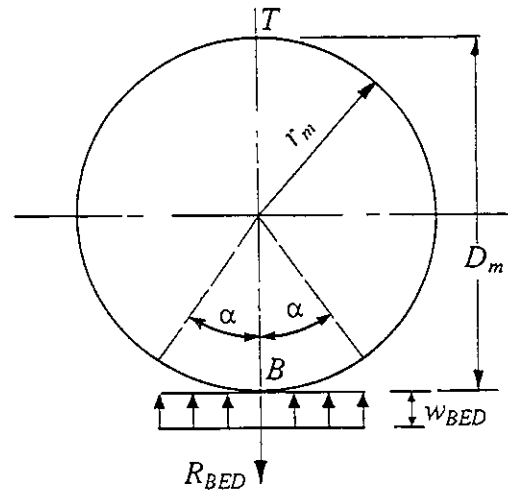

Figure 4. Paris' Case 7, Uniform Earth Load.

Figure 5. Paris' Case 8, Bedding Support Condition.

and the moment developed by the bedding support (Equation 13):

$$M_{FIELD} = M_{SOIL} + M_{BED} \tag{19}$$

$$M_{FIELD} = +0.147 W_E D_m - 0.038 w_{BED} D_m^2 \tag{20}$$

Substituting for w_{BED} using Equation 18 we get:

$$M_{FIELD} = +0.085 W_E D_m \tag{21}$$

This is the maximum moment at the invert for a buried pipe installed with a Class B bedding under a prism load with no lateral pressure.
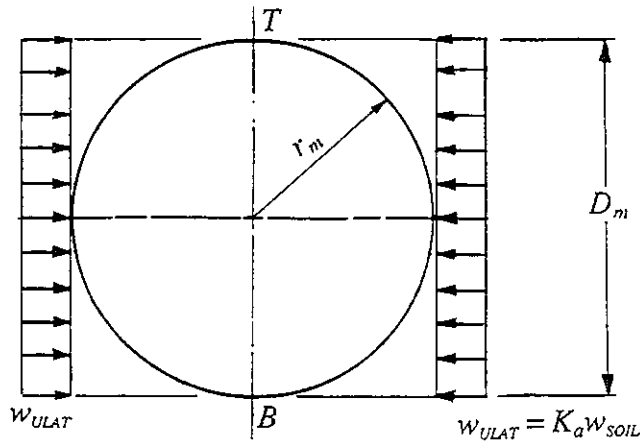
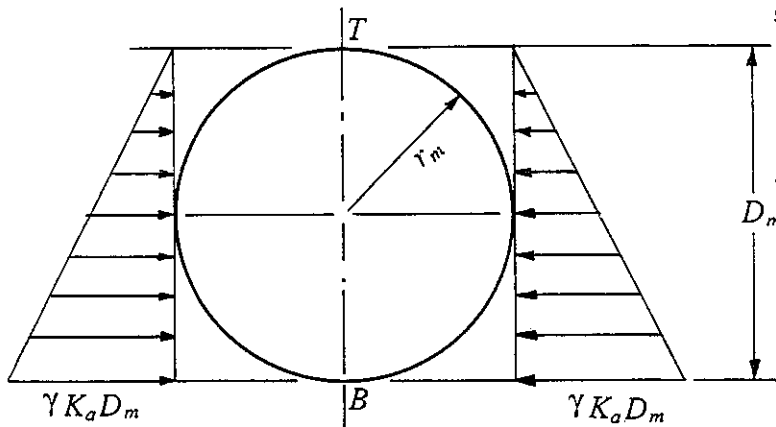Figure 6. Paris' Case 14, Uniform Lateral Pressure.

$$w_{ULAT} = K_a w_{SOIL}$$

Figure 7. Paris' Case 15, Triangular Lateral Pressure.

Buried Pipe - Uniform Lateral Pressure

The maximum moment developed in a situation where the backfill exerts a uniform lateral pressure on the pipe can be calculated based upon Paris' Case 14 (*Figure 6*). Conservatively assuming that the pipe does not deflect but experiences an active lateral earth pressure, the magnitude of this uniform lateral pressure, w_{ULAT}, is:

$$w_{ULAT} = K_a w_{SOIL} \tag{22}$$

Therefore, the maximum moment at the invert is:

$$M_{ULAT} = -0.063 K_a w_{SOIL} D_m^2 \tag{23}$$

Rearranging Equation 11 for w_{SOIL}, the vertical uniform earth pressure at the top of the pipe:

$$w_{SOIL} = \frac{W_E}{D_m} \tag{24}$$

Substituting into Equation 23 we get the maximum moment at the invert due to uniform lateral pressures:

$$M_{ULAT} = -0.063 K_a W_E D_m \tag{25}$$

Assuming that the backfill consists of common cohesionless material with $K_a = .33$:

$$M_{ULAT} = -0.021 W_E D_m \tag{26}$$

Buried Pipe - Triangular Lateral Pressure

Normally, vertical and lateral earth pressures increase linearly with the depth below the surface. This is modeled by Paris' Case 15 (*Figure 7*) for triangular lateral earth pressure where the maximum moment at the invert is:

$$M_{TLAT} = -0.292 K_a \gamma r_m^3 \tag{27}$$

since:

$$r_m = \frac{D_m}{2}$$

$$M_{TLAT} = -0.037 K_a \gamma D_m^3 \tag{28}$$

where:

γ = unit weight of backfill material, pounds per cubic foot

Assuming that the magnitude of the lateral pressure is limited to Rankine active earth pressure where:

$$K_a = 0.33$$

then:

$$M_{TLAT} = -0.012 \gamma D_m^3 \tag{29}$$

BEDDING FACTOR CALCULATIONS

Using the maximum moments it is possible to develop appropriate bedding factors for various loading conditions. As stated previously, the ratio of the moment developed in the three-edge bearing test to the calculated moment developed in the buried condition (M_{FIELD}) is the the bedding factor:

$$B_f = \frac{M_{TEB}}{M_{FIELD}} \tag{1}$$

Bedding Factor - No Lateral Pressure

To determine the bedding factor (B_f) for the case where there is no lateral earth pressure compare the maximum moment in a pipe subject to a T.E.B. test (Equation 9) to the field moment calculated for a buried pipe under a prism load with no lateral pressure (Equation 21).

$$M_{TEB} = M_{FIELD}$$
$$+ 0.159Q\, D_m = + 0.085 W_E\, D_m \qquad (30)$$

The ratio of the field load, W_E, to the T.E.B. load, Q, that will produce the same bending moment is the bedding factor:

$$B_f = \frac{W_E}{Q} \qquad (31)$$

Rearranging Equation 30 into this form produces a bedding factor of:

$$B_f = 1.87$$

This bedding factor for a trench installation with a Class B bedding (75°bedding angle) is recommended as 1.9 in past and current publications. As discussed previously, this conforms to Spangler's early work that ignores the effect of lateral earth pressures, and upon which current bedding factor values are based.

Bedding Factor - Uniform Lateral Pressure

Now, considering more realistic conditions, it is unlikely that a buried structure or pipe would not be subject to a lateral earth pressure from the backfill at the sides of the structure. This is particularly true with present day installation practices typically requiring 85 to 95% compaction of backfill along the sides of the pipe.

For this case consider only the lateral pressure induced as a result of the weight of fill over the top of the pipe. The bending moment developed with lateral pressure acting on the sides of a pipe was determined in Equation 25. This calculated maximum moment, based upon a uniform rectangular distribution of the lateral force, is:

$$M_{ULAT} = - 0.063 K_a W_E\, D_m \qquad (32)$$

The negative sign indicates that the moment due to the lateral pressure is in the opposite direction to the moment caused by the vertical load.

Using the principle of superposition, the maximum moment for a buried pipe installed with a Class B bedding experiencing both vertical and lateral pressures (combining Equations 21 and 25) is:

$$M_{FIELD} = (+ 0.085 - 0.063 K_a) W_E\, D_m \qquad (33)$$

Assuming a conservative Rankine active earth pressure coefficient of 0.33, the maximum moment is:

$$M_{FIELD} = (+ 0.085 - 0.021) W_E\, D_m \qquad (34)$$

$$M_{FIELD} = + 0.064 W_E\, D_m \qquad (35)$$

The effect of lateral pressure is to decrease the maximum moment at the pipe invert.

Equating the T.E.B. test moment to the field moment with both vertical and uniform lateral pressure:

$$M_{TEB} = M_{FIELD}$$
$$+ 0.159\, Q\, D_m = + 0.064\, W_E\, D_m \qquad (36)$$

The resulting bedding factor is:

$$B_f = \frac{W_E}{Q} = 2.48 \qquad (37)$$

Bedding Factor - Trapezoidal Lateral Pressure

The preceding case considered lateral pressure as a horizontal rectangular load which ignores the fact that lateral pressure increases from the top to the bottom of the pipe. Adding the triangular component from the pressure increase results in a trapezoidal configuration of lateral forces which further reduces the bending moment at the invert. The resulting bending moment can be calculated, using the principles of superposition, by combining the moments from vertical earth pressure with a Class B bedding (Equation 21), uniform lateral pressure (Equation 25), and triangular lateral pressure (Equation 28):

$$M_{FIELD} = + 0.085 W_E\, D_m - 0.063 K_a\, W_E\, D_m$$
$$- 0.037 K_a\, \gamma\, D_m^3 \qquad (38)$$

Assuming that only earth loads are applied to the pipe, the total earth load on top of the pipe (from Equation 11) is:

$$W_E = w_{SOIL}\, D_m = \gamma\, H D_m \qquad (39)$$

where:

$H =$ height of fill material above top of pipe, feet

Rearranging Equation 39 we get:

$$\gamma = \frac{W_E}{H D_m} \qquad (40)$$

Substituting this into Equation 38:

$$M_{FIELD} = + 0.085 W_E\, D_m - 0.063 K_a\, W_E\, D_m \qquad (41)$$
$$- 0.037\, \frac{K_a}{H}\, W_E\, D_m^2$$

$$M_{FIELD} = \left[0.085 - 0.063 K_a \right.$$
$$\left. - 0.037 K_a \left(\frac{D_m}{H} \right) \right]\, W_E\, D_m \qquad (42)$$

Using Equation 42, bedding factors can be determined for various combinations of K_a, D_m and H. For example, for an installation with a $K_a = 0.33$, $H/D_m = 2$, and a Class B bedding (an effective bedding angle of 75°) with trapezoidal lateral pressure distribution, the bedding factor is 2.73. This is a .25 increase when compared to the case considering only a horizontal rectangular distribution.

DEFLECTION EFFECTS

As discussed in the previous section on lateral pressure, active earth pressures are generated when the soil pushes against the wall of the pipe. With any deflection of the pipe under vertical load, the horizontal dimension increases resulting in the sides of the pipe pushing against the soil such that passive earth pressures are generated. This is particularly true when a high soil compaction level is used in backfilling around the pipe. As discussed previously, passive earth pressures can be much higher than active earth pressures.

Tables 2 and 3 list bedding factors calculated using various lateral earth pressure coefficients (K) to illustrate the effect of deflection and lateral earth pressure upon load carrying capacity. Since rigid concrete pipe typically experiences small deflections, low passive earth pressure coefficients are considered. In cases where high soil compaction levels are used around the pipe, small movements will generate full passive earth pressure and higher lateral earth pressure values can be considered. The lateral earth pressure coefficient (K) used in the tables can be either an active (K_a) or passive (K_p) earth pressure coefficient depending upon pipe deflection and installation conditions.

AXIAL THRUST

The lateral forces that reduce bending moment also cause an axial thrust component to occur in the wall of the pipe at the point where moment is maximum. This thrust produces compression which is a beneficial factor effectively increasing shear strength by reducing the maximum tension stresses in the pipe wall. Consideration of thrust, which is typically ignored, would further increase the bedding factor.

SUMMARY

The bedding factors typically used for concrete pipe are quite conservative, and significantly understate the real supporting strength of concrete pipe. Installations are generally specified with select or granular material compacted to 85-95% surrounding the pipe. Both the equipment to provide the compactive effort and the equipment to verify that such compaction has been achieved have improved dramatically since the research of the 1920's. The effects of lateral pressures and axial thrust resulting from today's installations should be considered in determining bedding factors for both trench and embankment conditions. Tables 2 and 3 present bedding factors calculated for typical beddings and various lateral pressure distributions.

Table I. Beddings.

Bedding Class	Bedding Dimensions	
	Width	Angle(degrees)
*	0.7 B_c	90
B	0.6 B_c	75
C	0.5 B_c	60
D	Flat Subgrade	Line Loading

NOTES:
B_c = outside diameter of the pipe
* Shown to illustrate true 90 degree bedding, does not conform to a current bedding class

Table 2. Bedding Factors, Uniform Lateral Pressure.

Bedding		Effective Bedding Angle 2α (degrees)	Bedding Factors (B_f)			
Class	Width		$K = 0$	$K = 0.33$	$K = 0.50$	$K = 1.0$
*	$0.7\,B_c$	90	2.01	2.73	3.35	9.94
B	$0.6\,B_c$	75	1.87	2.48	2.97	7.23
C	$0.5\,B_c$	60	1.69	2.17	2.54	5.13

NOTES:
B_c = outside diameter of the pipe
K = coefficient of lateral earth pressure, can be either active or passive earth pressure
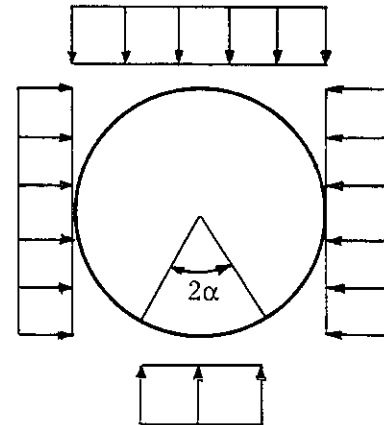*Shown to illustrate true 90 degree bedding, does not conform to a current bedding class

Table 3. Bedding Factors, Trapezoidal Lateral Pressure.

Bedding		Effective Bedding Angle 2α (degrees)	$\dfrac{H}{D_m}$	Bedding Factors (B_f)	
Class	Width			$K = 0.33$	$D = 0.50$
*	$0.7\,B_c$	90	1.00	3.46	5.48
			1.33	3.24	4.73
			2.00	3.05	4.16
			4.00	2.88	3.71
			10.00	2.79	3.48
B	$0.6\,B_c$	75	1.00	3.06	4.54
			1.33	2.89	4.02
			2.00	2.73	3.59
			4.00	2.60	3.25
			10.00	2.52	3.08
C	$0.5\,B_c$	60	1.00	2.61	3.61
			1.33	2.48	3.27
			2.00	2.37	2.99
			4.00	2.27	2.75
			10.00	2.21	2.62

NOTES:
B_c = outside diameter of the pipe
K = coefficient of lateral earth pressure, can be either active or passive earth pressure
*Shown to illustrate true 90 degree bedding, does not conform to a current bedding class