CONCRETE PIPE INFORMATION CONCRETE PIPE

CP Info

american concrete pipe association ▲ 8320 old courthouse road ▲ vienna, virginia 22180

# CULVERT VELOCITY REDUCTION WITH AN OUTLET EXPANSION

*This article describes the concept of using an abrupt expansion at the outlet end of a concrete culvert for purposes of outlet velocity reduction. The method is intended for culvert operating under conditions of outlet control at maximum discharge. A model test program was set up to establish the design requirements for the expansion, and experimentally verify the performance. The results of this research are also presented.*

## GENERAL

There are many culvert applications wherein the designer specifies maximum outlet velocities, usually in order to simplify problems of possible scour and erosion at the outlet. For example, the velocity may be specified at 6 to 8 feet per second, which velocity is considered sufficiently low that scour can be easily resisted at the outlet with riprap, natural till, or perhaps a grassed channel. Under conditions of outlet control with the pipe flowing full, the required pipe size is then determined from the continuity equation as

$$D = \sqrt{\frac{4Q}{\pi V}} \qquad (1)$$

in which D is the pipe diameter in ft, Q is the maximum (design) discharge in cubic feet per second, and V is the pipe velocity in feet per second.

Unfortunately, when the design is governed by limiting maximum outlet velocity, concrete pipe may lose the advantage of a size differential over other pipe which is less efficient hydraulically. Using the method proposed here, however, a more economical design is possible. The method consists simply of using an abrupt expansion at the outlet end of the culvert. This can be achieved in practice by telescoping one length of larger pipe at the outlet over the preceding pipe, as shown in Figure 1. In this way a smaller diameter can be used for most of the pipe, and the outlet velocity specification can still be met. The purpose of using an abrupt expansion as opposed to a gradual expansion is that the former can be accommodated quite simply using existing sizes of precast concrete pipe, and fabrication of a special transition section is unnecessary.

The proposed design applies only to outlet control situations where the tailwater is at or above the crown of the pipe at design flow. Downstream submergence of this type is fairly common in flatland drainage problems, or in multi-pipe installations where smaller diameters are used because of limited headroom. Downstream submergence is particularly prevalent on irrigation projects where, under controlled tailwater situations, the conduit is deliberately set to insure full flow and thereby utilize the full area of the pipe.

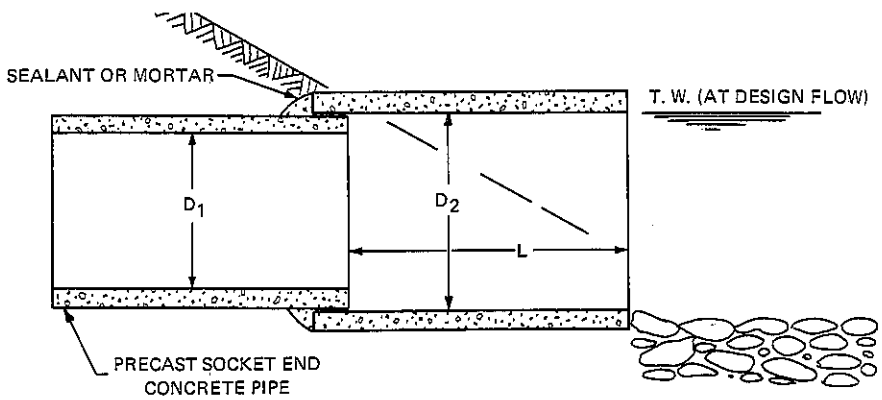


**FIGURE 1. DEFINITION SKETCH FOR ABRUPT EXPANSION**

## THEORY FOR ABRUPT EXPANSION

In the absence of an expansion the head lost at the outlet will be equal to the conduit velocity head.

$$h_{L1} = V_1^2/2g \qquad (2)$$

With an expansion, in which the velocity is reduced to $V_2$, the outlet loss will be

$$h_{L2} = V_2^2/2g \qquad (3)$$

In addition there will be a head loss due to the abrupt expansion, as given by the well established expression

$$h_{L3} = (V_1 - V_2)^2/2g \qquad (4)$$

The net head gain $h_g$ due to the expansion will be $h_{L1}-(h_{L2}+h_{L3})$, or

$$h_g = \frac{V_1^2 - V_2^2}{2g} - \frac{(V_1 - V_2)^2}{2g} \qquad (5)$$

Equation (5) may be reduced to

$$h_g = \frac{V_2\,(V_1 - V_2)}{g} \qquad (6)$$

For a circular pipe equation (6) may be written as

$$h_g = \frac{16\,Q^2}{g\,(\pi D_1^2)^2} D_r^2\,(1 - D_r^2) \qquad (7)$$

in which $D_r$ is the diameter ratio $D_1/D_2$. Equation (7) shows that there will be no head gain if $D_r=1$ (no expansion) or if $D_r=0$ (infinitely large expansion). The maximum possible head gain may be found by setting $dhg/dD_r=0$, for which $D_r=0.707$. In this case $D_2=1.414D_1$ and the head gain would be $0.5\ V_1/2g$.

Because of the extra pipe length required to achieve a proper expansion with $D_r=0.707$, this diameter ratio would not be most economical in practice. Fortunately, most of the benefit can be obtained with a larger value of $D_r$. For example, with $D_r=0.85$, the head gain will still be 80% of maximum. Considering also that scour potential of the jet varies as velocity squared, then for a given discharge the scour potential will vary as the fourth power of the reciprocal of the outlet diameter. In this case a value of $D_r=0.85$ would reduce the scour potential to almost half.

It is important to note that the head gained at the inlet and velocity reduction at the outlet go together. One cannot be obtained without the other. If the outlet velocity is to be successfully reduced the head must be gained whether it is needed or not.

It is evident that the effectiveness of the expansion will depend upon the length of pipe following the abrupt expansion. This length is shown as L on Figure 1. If L is shorter than the length necessary for a complete expansion of the flow, the head gain will be less than given by equation (7) and the outlet velocity will be greater than calculated from $V=Q/A$. Further, the required length will depend upon $D_r$, in that to achieve full expansion smaller values of $D_1/D_2$ will require larger values of $L/D_1$.

The efficiency of the expansion may be designated as $C_r$, called the recovery coefficient, which is the ratio of the actual head gain to the theoretical head gain given by Equation (7). Hence,

$$C_r = \frac{h_g\, g(\pi D_1^2)^2}{16Q^2\, D_r^2(1 - D_r^2)} \qquad (8)$$

The object of the experimental program was to determine the relationship between $C_r$ and $L/D_1$ for a given value of $D_r$.

Standard pipe size increments are 3 inches in the smaller sizes and 6 inches in the larger sizes. Values of $D_r$ for various standard size ratios are given in Table 1. Obviously, considerable time and expense would be involved to test each individual size ratio. Fortunately, the value of $D_r$ does not change greatly for the group so it was felt that considerable useful data could be obtained with tests based on the average $D_r$ for the group.

## MODEL TEST AND RESULTS

The test model consisted of an 8½-foot long smooth acrylic pipe with an inside diameter of 0.330 feet. An expanded section with an inside diameter of 0.374 could be added to the end, thus permitting a test value for $D_r$ of 0.883. The expanded section was telescoped over the smaller pipe and could be adjusted to change $L/D_1$. The bed of the discharge channel was placed at the same elevation as the invert of the outlet pipe.

Discharge from a metered supply was passed through the pipe. The tailwater was adjusted so that the crown of the pipe was just submerged. Initially, the pipe was tested without any expansion over a range of discharges, and later the same procedure was followed with various expansions added to the end. The difference in head across the pipe was noted in each case. Scour tests at the outlet were also used to test the effectiveness of the expansion.

For purposes of data analysis the results were plotted nondimensionally, as shown in Figure 2. The value H is the measured differential head (headwater minus tailwater) across the conduit. The upper curve gives $H/D_1$ versus $Q/D_1^{5/2}$ for the pipe with no expansion on the end. The lower curves give $H/D_1$ with various values of $L/D_1$. The vertical

**TABLE 1**

VALUES OF $D_r$ FOR VARIOUS STANDARD PIPE SIZE DIFFERENTIALS

| Pipe Sizes | $D_r$ |
|---|---|
| 18/21 | 0.858 |
| 21/24 | .875 |
| 24/27 | .889 |
| 27/30 | .900 |
| 30/33 | .908 |
| 36/42 | .858 |
| 42/48 | .875 |
| 48/54 | .889 |
| 54/60 | .900 |
| 60/66 | .908 |
| | Average 0.886 |

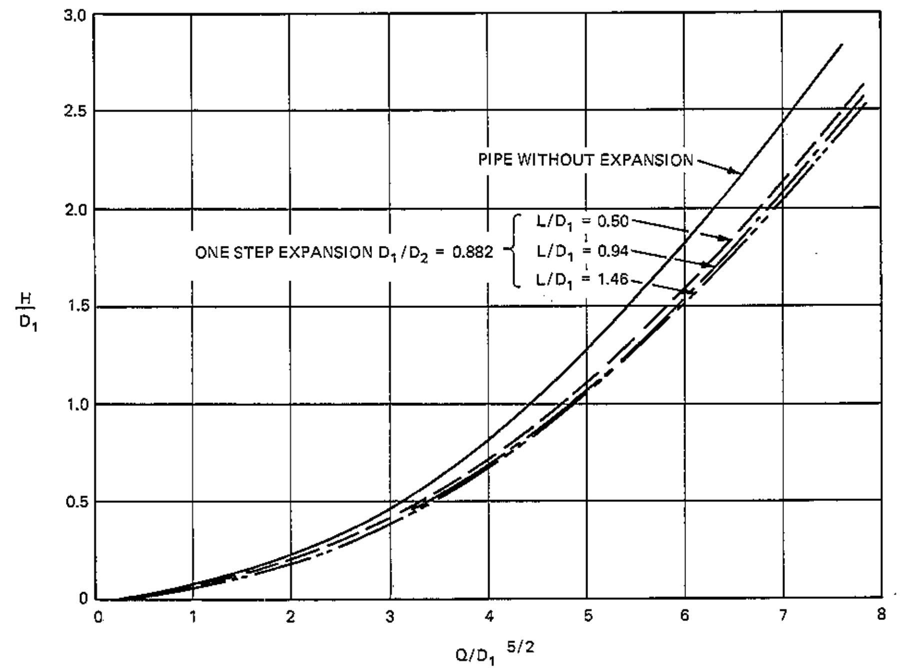


**FIGURE 2. DIMENSIONLESS PLOT OF HEAD VERSUS DISCHARGE**

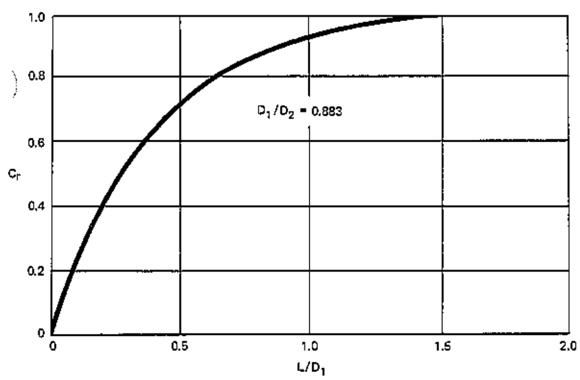


**FIGURE 3. RECOVERY COEFFICIENT VERSUS EXPANSION LENGTH FOR ABRUPT EXPANSION**

difference between the upper curve and a lower one gives $h_g/D_1$ for the outlet condition represented by the lower curve. The data were carefully plotted to a large scale and the values of $h_g/D_1$ were read directly from the plot. The results of this process are given in Table 2.

The plot of recovery coefficient versus $L/D_1$ from the data in Table 2 is shown in Figure 3, and indicates that the expansion is 100% efficient at $L/D_1 = 1.5$. However, it is suggested that $L/D_1 \geqq 1.0$ would be satisfactory for design purposes for the size differentials listed in Table 1. An $L/D_1 = 1.0$ would still account for 90% of the head gain. Theoretically, $L/D_1$ should be slightly greater than unity for the $D_r$ values less than 0.883 in Table 1, and could be slightly less than unity for the $D_r$ values greater than 0.883.

A $D_1/D_2$ ratio of 0.775, simulating a greater size differential, was also tested, as reported elsewhere [2]. Indications were that an expansion length of 4 pipe diameters would be required to properly achieve the desired expansion and velocity reduction. This is not surprising when it is considered that for $D_1/D_2 = 0.5$ and $L/D_1 = 8$ is required to achieve a complete expansion, as shown by Chatervedi [1]. Data reported here is confined to the case of $D_r = 0.883$ because this size differential can be accommodated in most field cases by the addition of a single length of the larger pipe.

## SCOUR TESTS

In a design in which outlet velocities govern, the head saved by using an expansion may be of secondary importance. Velocity reduction is the primary requirement. As previously indicated, however, if the velocity is reduced then head is saved automatically, and the latter may be taken as proof of the former. Nonetheless, scour tests were carried out as further proof of the effectiveness of the expansion.

Scour comparisons were made for the pipe with and without the

**TABLE 2.**

RECOVERY COEFFICIENT FOR $D_1/D_2 = 0.883$

| | | $L/D_1 = 1.46$ | | $L/D_1 = 0.94$ | | $L/D_1 = 0.50$ | |
|---|---|---|---|---|---|---|---|
| $Q/D_1^{5/2}$ | Theoretical $h_g/D_1$ (Eq. 7) | Experimental $h_g/D_1$ (Figure 2) | $C_r$ | Experimental $h_g/D_1$ (Figure 2) | $C_r$ | Experimental $h_g/D_1$ (Figure 2) | $C_r$ |
| 3 | 0.079 | 0.081 | 1.03 | 0.073 | 0.924 | 0.056 | 0.710 |
| 4 | .140 | .140 | 1.00 | .130 | .929 | .100 | .715 |
| 5 | .219 | .221 | 1.01 | .200 | .913 | .160 | .730 |
| 6 | .315 | .305 | 0.97 | .280 | .890 | .225 | .715 |
| 7 | .429 | .420 | 0.98 | .380 | .888 | .310 | .723 |
| | | Average $C_r = 1.00$ | | Average $C_r = 0.91$ | | Average $C_r = 0.72$ | |

expansion, using a stone bed of median size $d_m = 0.02$ feet. Figure 4 shows the typical appearance of the elongated saucer shaped scour pattern which formed in the bed downstream from the pipe outlet. A plan view of the scour pattern corresponding to a $Q/D_1^{5/2} = 5$ is shown in Figure 5. The numbers on the contours represent scour depth in feet below the invert elevation.

With an expansion of $D_1/D_2 = 0.883$ added to the outlet, it was necessary to increase the discharge by 20%, to $Q/D_1^{5/2} = 6$, to produce similar scour, as shown in Figure 6.

Scour data was plotted non-dimensionally as shown in Figure 7. The intercept on the abscissa represents the limiting maximum value of $Q/D_1^{5/2}$ for no scour in that particular size of bed material. With no expansion the value is 2.7 and with the $D_r = 0.883$ expansion the value is 3.5. Scour should commence at approximately the same velocity in either case, and this is essentially verified by the fact that $2.7/(0.883)^2 \approx 3.5$.

## CONCLUSION

1. Head may be gained and outlet velocities reduced by the addition of an abrupt expansion to the end of a conduit designed to operate under outlet control.
2. An expansion length of $L = D_1$ is sufficient to recover 90% of the theoretical head gain for $D_r = 0.883$. This diameter ratio conforms closely to a one size differential in standard sizes of concrete pipe.
3. The theoretical head gain for the abrupt expansion is

$$h_g = \frac{V_2 (V_1 - V_2)}{g}$$

4. The velocity reduction aspect of the expansion has been verified by scour tests.


*REFERENCES*

[1] *Chatervedi, M. C., "Flow Characteristics of Axisymmetric Expansions," Journal of the Hydraulics Division, ASCE, May, 1963.*

[2] *Smith, C. D., "Expansions at Conduit Outlets," Canadian Hydraulics Conference, Edmonton, Canada, May, 1973.*


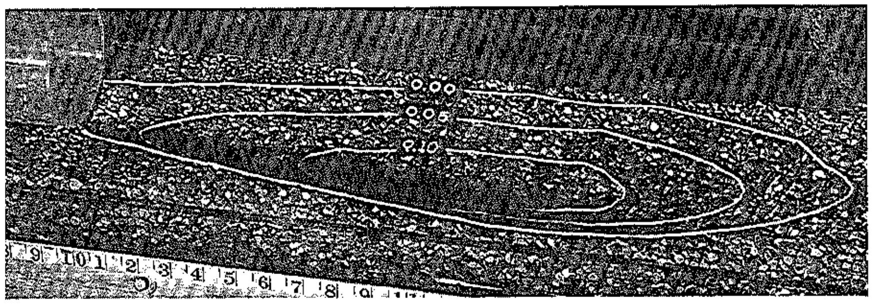


**FIGURE 4. TYPICAL SCOUR PATTERN AT CONDUIT OUTLET**

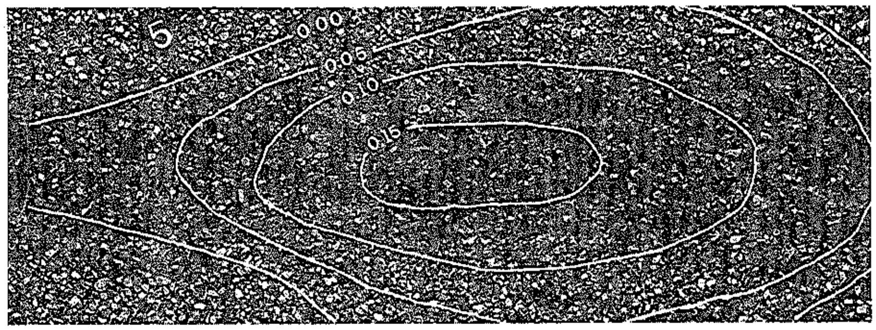


**FIGURE 5. SCOUR PATTERN CONTOURS FOR SINGLE PIPE**
$Q/D_1^{5/2} = 5$; $d_m/D_1 = 0.058$

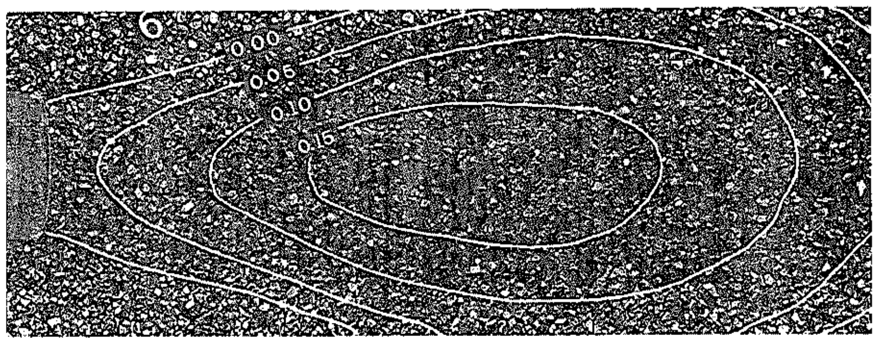


**FIGURE 6. SCOUR PATTERN CONTOURS FOR PIPE WITH EXPANSION** $D_1/D_2 = 0.883$; $L/D_1 = 1$; $Q/D_1^{5/2} = 6$; $d_m/D_1 = 0.058$

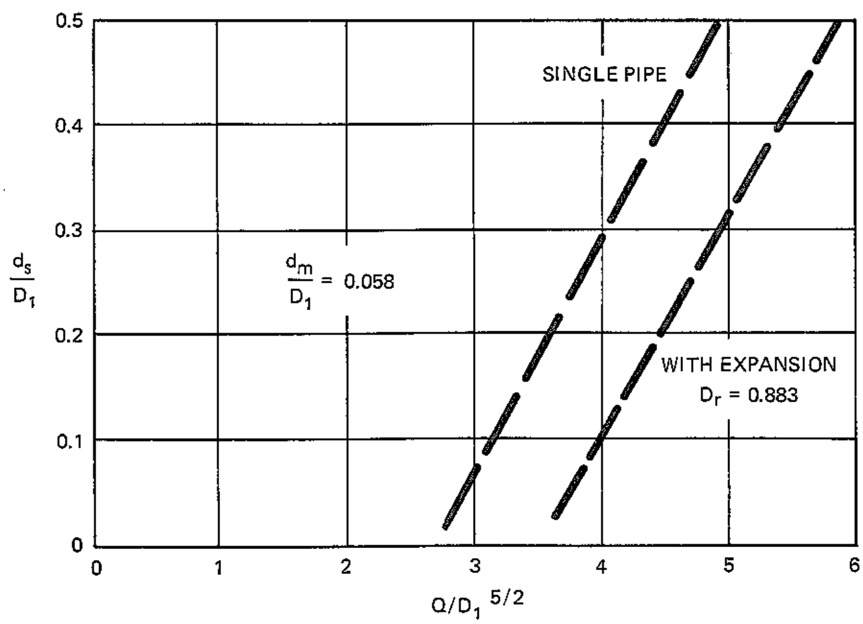


**FIGURE 7. NON-DIMENSIONAL REPRESENTATION OF SCOUR**


Cp Info 3R7910



by C. D. Smith, Professor of Civil Engineering
University of Saskatchewan, Saskatoon, Canada