CP Info
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CULVERT VELOCITY REDUCTION BY INTERNAL ENERGY DISSIPATORS

The designer is often concerned with possible scour or erosion at the outlet of a drainage culvert constructed on a steep slope. Such erosion can cause serious maintenance, silting and pollution problems. The high velocity associated with flow on steep slopes is the critical parameter in the erosion process.

Reduction of the velocity of such flows is generally accomplished by the formation of a hydraulic jump. A hydraulic jump converts shallow, high velocity flow to deeper, low velocity flow while losing considerable energy in the resulting turbulence. Most outlet protection devices are essentially stilling basins, designed so the hydraulic jump is formed in the basin.

This article describes dissipators intended to form the hydraulic jump within the culvert, thus eliminating costly outlet structures. These dissipators are cirular rings spaced along the pipe at the downstream end. The rings cause a series of hydraulic jumps to form in the barrel of the pipe, resulting in a near optimum dissipation of energy and virtually minimum possible total energy at the outlet.

GENERAL

Previous research conducted at Virginia Polytechnic Institute on the use of roughness elements in open channels established that excess energy in storm water flowing down steep drainage channels could be dissipated by constructing roughness elements within the channel. Since culverts operating under inlet control simulate open channel flow, application of this type of internal energy dissipation to culverts could possibly result in more efficient utilization of the culvert barrel and reduced outlet velocities.

In August, 1969, the American Concrete Pipe Association contracted with Virginia Polytechnic Institute and State University (VPI) to investigate and determine the feasibility and applicable design procedures for using roughness elements as energy dissipators of free-surface flow in circular concrete pipe culverts. Results of the research are published in Highway Research Record Number 373 **Roughness Elements as Energy Dissipators of Free-Surface Flow in Circular Pipes.** Because of the criteria of assuring free surface flow, full capacity of the conduit was not realized and necessitated an increase in pipe size within the length of culvert in which the roughness elements are placed. Based on the laboratory and field observations during this initial research, subsequent tests were conducted for full flow conditions occurring near the outlet end at maximum design discharge. By eliminating the criteria of free surface flow and allowing the culvert

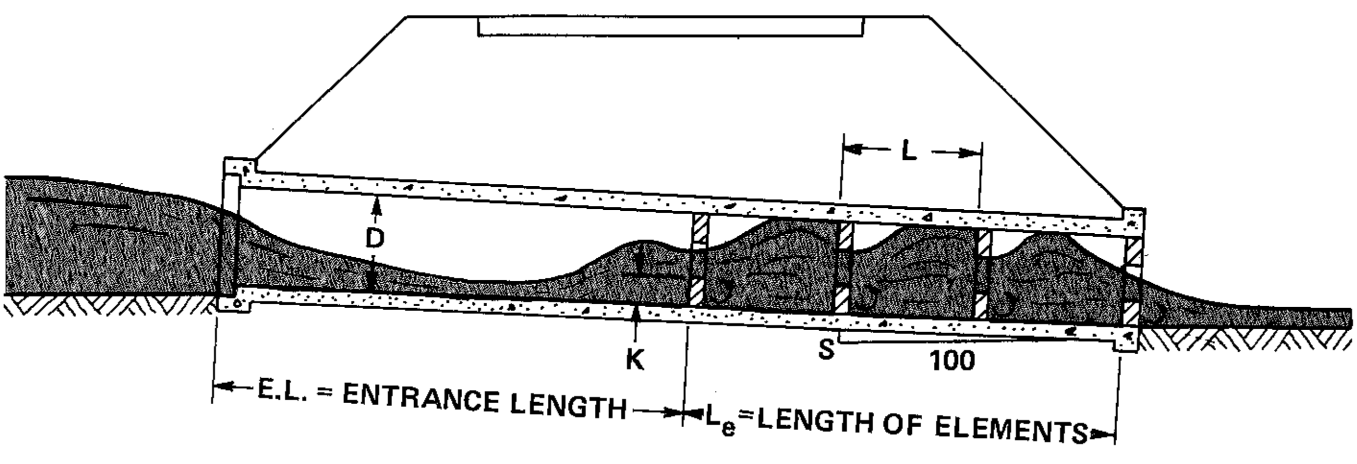


FIGURE 1. TUMBLING FLOW IN PIPE CULVERT.

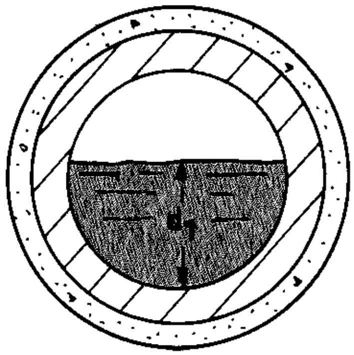


FIGURE 2. ROUGHNESS ELEMENT IN PIPE.

to approach full flow, it was found velocity reduction could be effected without an increase in pipe size. The results of this later research and design procedures for both the full flow condition and the free surface flow condition are presented in the following paragraphs.

FREE SURFACE FLOW TESTS

The performance characteristics of dissipator rings were investigated initially in laboratory models and later with a full scale 18-inch reinforced concrete pipe prototype. Different numbers of rings of various cross-sectional dimensions and spacings were tested in the 6-inch diameter clear plastic model pipe which could be adjusted to any slope from zero to 30 percent. An early conclusion was that only five rings were necessary to achieve consistent results. The full scale prototype was tested at flatter slopes than the laboratory model because test facilities with unlimited quantities of water were not available within a reasonable distance of VPI.

Since the objectives of the research were to dissipate energy and reduce high velocities associated with culverts on what are considered steep slopes, the culverts were operating under inlet control. Accordingly, the flow characteristics were observed to be one of critical flow at the entrance of the pipe with the flow accelerating down the length of the pipe until the first ring, or roughness element, was reached. At that point, a hydraulic jump was formed, with extreme turbulence. The flow then encountered another roughness element while still in an agitated condition from the first and this pattern of action was repeated until a cyclic condition was reached, where the flow conditions over the roughness elements were uniform. Generally, this cyclic action was attained after the second or third element. The agitated flow, characterized by a greater depth over the element than before it, a fall into a *valley* between the elements, and a form resembling a hydraulic jump shortly before the next element, is called *tumbling* flow. Thus one cycle is completed and the flow *tumbles* into the next cycle until the outlet is reached. This *tumbling* flow can only be established and maintained under less than full flow conditions. Figure 3 shows how tumbling flow with a free flow surface at less than maximum design discharge appeared in the 6-inch clear plastic pipe.

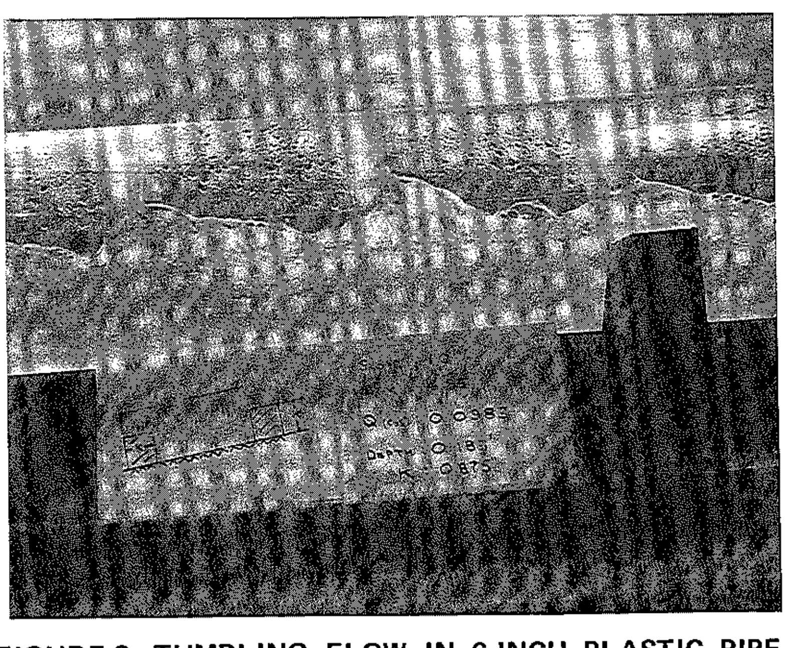


FIGURE 3. TUMBLING FLOW IN 6-INCH PLASTIC PIPE.

FULL FLOW TESTS

During the previous VPI research on open channel flow, it was observed that if one large dissipator element was placed upstream it created a large hydraulic jump which was maintained by the smaller downstream elements. In applying this observation to pipe flow at maximum design discharges, it was theorized that the hydraulic jump at the large upstream ring would cause the pipe to flow full with the smaller downstream rings maintaining the full flow condition.

Several tests of various ring configurations quickly indicated the soundness of this approach. Subsequently extended tests for the full flow condition were made in the laboratory model with a ring configuration consisting of three small rings at the exit preceded by one large ring at double spacing as illustrated in Figure 4. The three small rings were spaced at spacing-diameter ratio (L/D) of 1.5 with a ring height-diameter ratio (K/D) of 0.0625. The large ring, at double spacing, had a height ratio K/D of 0.146.

Model tests were run for this configuration at three slopes of 4.3, 9.3 and 15.2 percent. In order to compare these model tests with the full scale prototype tests under *free surface flow,* the range of test flows was equivalent to 10 to 15 cubic feet per second in an

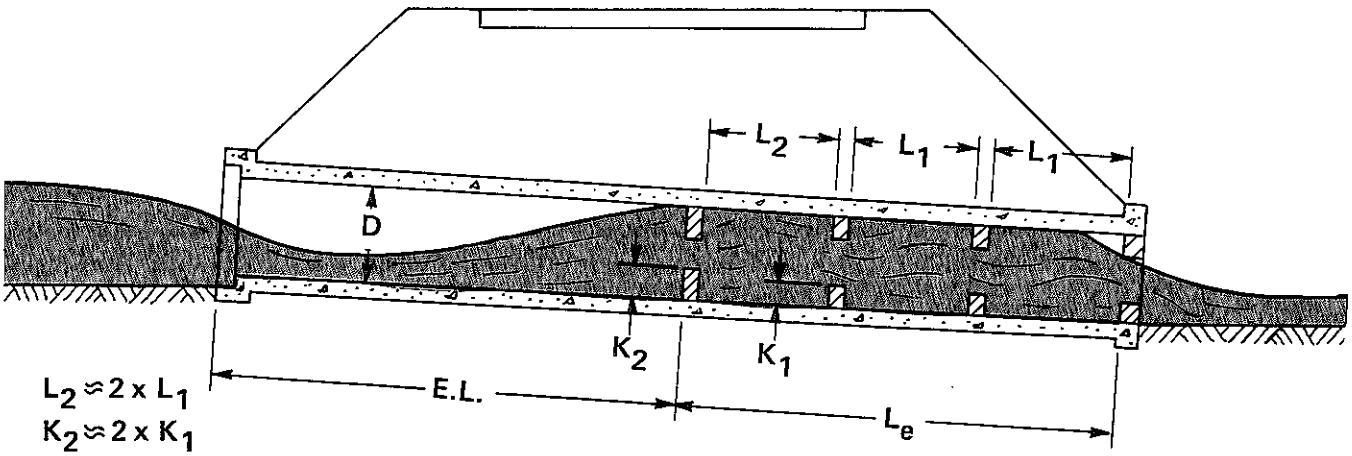


FIGURE 4. FULL FLOW IN PIPE CULVERT.

18-inch diameter pipe. In all of these larger flows (larger than indicated by tumbling flow criteria), the pipe flowed full at the outlet with the initial hydraulic jump varying in position above the leading ring depending upon the slope and flow rate. In some cases there were slugs of air moving unsteadily down the pipe, entering at a vortex in the headwater and moving through regions of full flow in entrained bubbles. In such cases, the quantity and movement of air through the pipe would indicate pressures only slightly above atmospheric and inlet control still governed. Table I details the test data and results and Table II lists the computations relating the test results to the expected performance of an 18-inch diameter prototype pipe. The prototype discharge Q_p was determined by using a Froude relationship for similitude, $Q_r = L_r^{5/2}$. In all cases except where tumbling flow is noted, the model pipe was flowing full at the downstream end. Therefore, prototype velocity Q_p was determined by dividing prototype discharge by prototype area, where the prototype area is the area of the pipe at the outlet minus the decrement in area resulting from the last ring.

TABLE I. TEST RESULTS OF 6-INCH PLASTIC PIPE.

Test Number	Slope (percent)	Q_{model} (cfs)	HW (inches)	$Q_{prototype}$ (cfs)	Remarks
1	15.2	0.946	7.00	14.7	Cavity from ring to approximately 10 feet upstream from first ring.
2	15.2	0.825	7.00	12.9	Cavity from ring to approximately 1 foot upstream from first ring.
3	15.2	0.676	6.00	10.5	Some air entrainment in ring area.
4	15.2	0.509	–	7.9	Verge of tumbling flow.
5	9.3	0.527	–	8.2	Verge of tumbling flow.
6	9.3	0.676	6.75	10.5	–
7	9.3	0.825	7.50	12.9	Cavity from 5 feet to 15 feet upstream from first ring.
8	9.3	0.946	8.00	14.7	Essentially full, some unsteady slugs of air.
9	4.3	0.858	15.00	13.4	Full.
10	4.3	0.769	10.25	12.0	Essentially full, some unsteady slugs of air.
11	4.3	0.676	7.75	10.5	Slugs of fluid progressing upstream to entrance, jump in front of ring.
12	4.3	0.588	6.50	9.2	Cavity from 6 feet to 1 foot, intermittent.
13	4.3	0.482	6.00	7.5	Tumbling flow.

TABLE II. CALCULATED PERFORMANCE OF 18-INCH CONCRETE PIPE.

Test Number	$Q_{prototype}$ (cfs)	V = Q/A (fps)	Q_{full} (cfs)	$\frac{Q_{prototype}}{Q_{full}}$	$\frac{V_{prototype}}{V_{full}}$	Percent Velocity Reduction
1	14.7	9.90	46	0.32	0.88	56.8
2	12.9	8.69	46	0.28	0.86	61.2
3	10.5	7.07	46	0.23	0.82	66.8
4	7.9	–	46	0.17	0.75	–
5	8.2	–	35	0.23	0.82	–
6	10.5	7.07	35	0.30	0.87	59.4
7	12.9	8.69	35	0.37	0.93	53.3
8	14.7	9.90	35	0.42	0.95	47.9
9	13.4	9.03	23	0.58	1.03	34.1
10	12.0	8.09	23	0.52	1.01	39.6
11	10.5	7.07	23	0.46	0.98	45.6
12	9.2	6.20	23	0.40	0.94	50.4
13	7.5	–	23	0.33	0.89	–

FULL FLOW DESIGN PROCEDURE

Based on the preceding full flow test results, the following design procedure is suggested:

1. Select required pipe size based on the hydraulic design procedures presented in Hydraulic Engineering Circular Nos. 5 or 10 prepared by the Federal Highway Administration or the Concrete Pipe Design Manual published by the American Concrete Pipe Association.
2. For culverts operating under inlet control, determine outlet velocity by means of Manning's Formula.
3. If velocity reduction is desired, select a roughness element size for the three downstream rings with a height-diameter ratio between 0.06 and 0.09,

   $$0.06 \leqq K/D \leqq 0.09$$

   and a spacing-diameter ratio of 1.5. $L/D = 1.5$

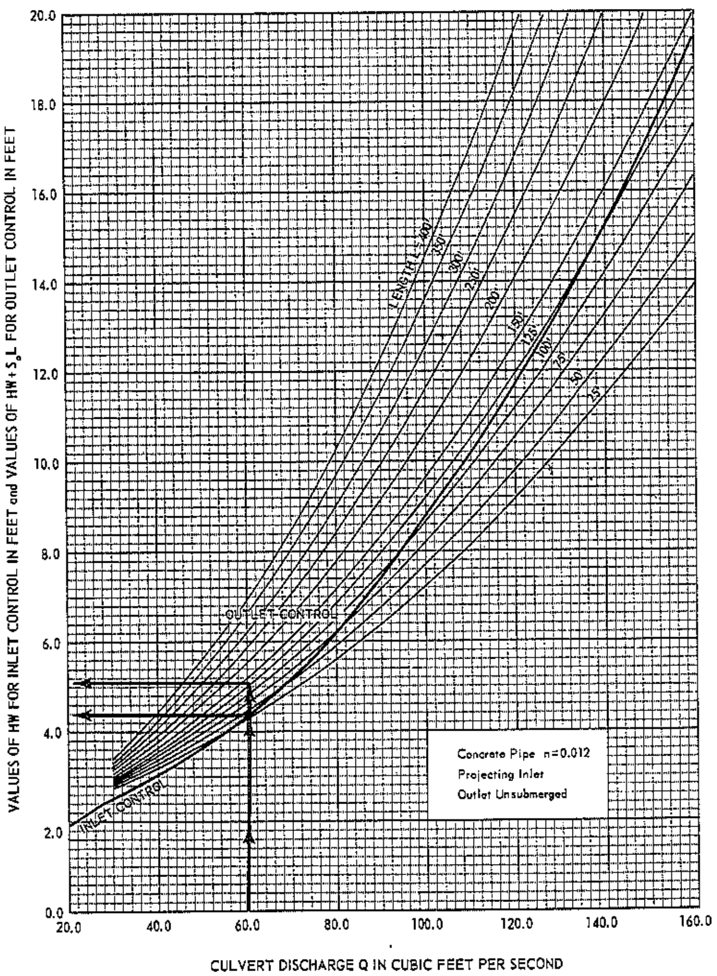
   The single upstream ring would then be located at twice this spacing and sized to be approximately double the downstream rings.
4. Determine the hydraulic cross-sectional area at the last downstream ring.
5. Divide the design discharge by the resultant area determined in Step 4 to determine the outlet velocity.

Example 1.

Given: Culvert, 36-inch diameter, 125 feet long, n = 0.012, 4% slope
Design Q = 60 cfs
AHW = 4.5 feet

Find: Size and spacing of roughness elements for full flow conditions

FIGURE 44 CULVERT CAPACITY 36-INCH DIAMETER PIPE



Solution:

1. Check culvert control.
   Figure 44, p. 222 Concrete Pipe Design Manual

   Inlet control:
   HW = 4.4 feet o.k.
   Outlet control:
   $HW + S_oL = 5.1$ feet
   $HW = 5.1 - 0.04 \times 125$
   HW = 0.1 feet o.k.

   Therefore, Inlet Control governs.

2. Determine outlet velocity
   Figure 4, p. 181 Concrete Pipe Design Manual

   $Q_{full} = 145$ cfs
   $V_{full} = 20.5$ fps

   Figure 18, p. 195 Concrete Pipe Design Manual

   $Q_d/Q_{full} = 60/145 = 0.41$
   $V_d / V_{full} = 0.94$
   $V_d = 0.94 \times 20.5 = 19.3$ fps

3. Velocity reduction desired.
   Downstream ring height
   $0.06 \leqq K/D \leqq 0.09$

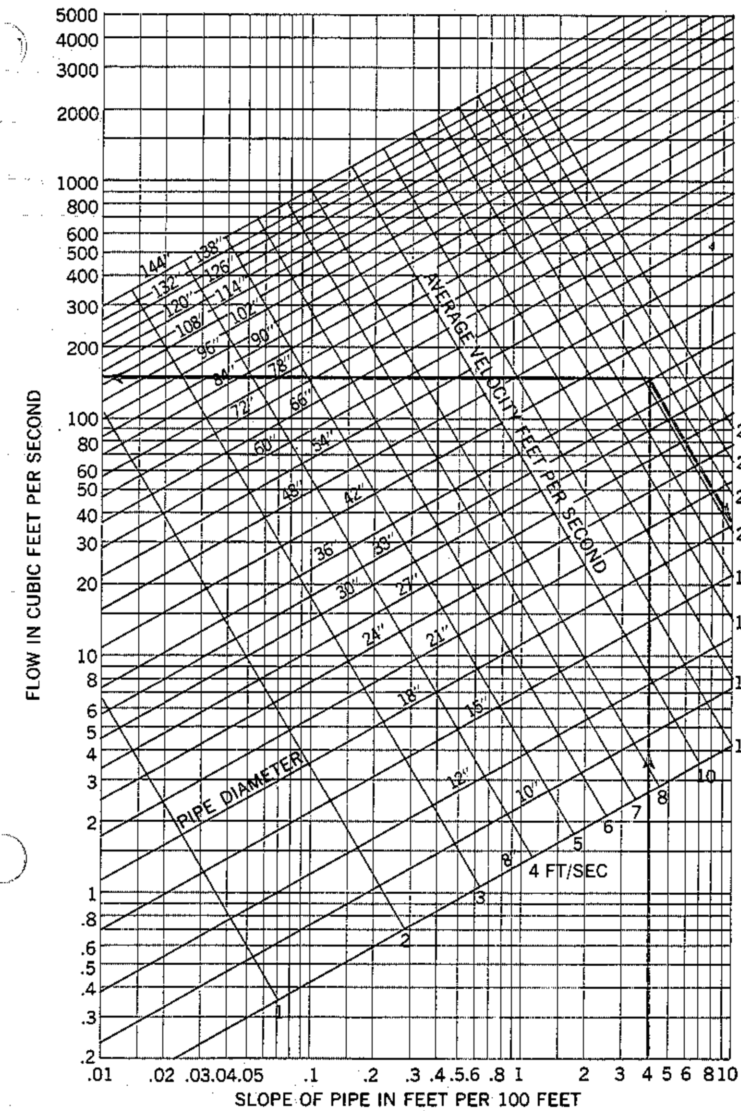
   Use K = 0.25 feet or 3 inches

   Downstream ring spacing
   L/D = 1.5
   Use L = 4.5 feet

   Upstream ring height
   Use K = 6 inches

   Upstream ring spacing
   Use L = 9 feet

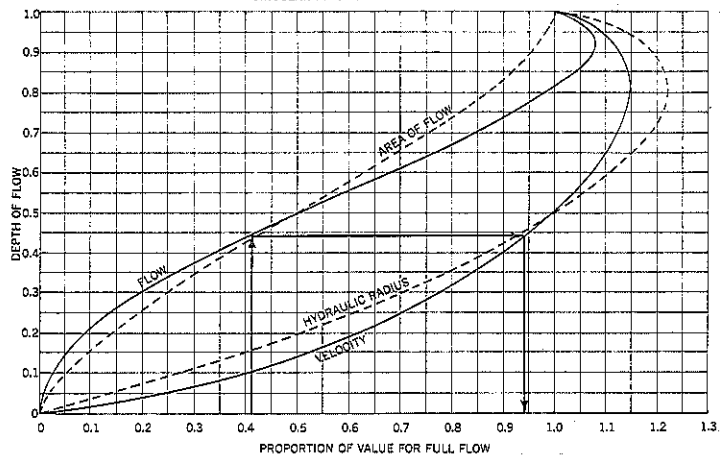
FIGURE 4 FLOW FOR CIRCULAR PIPE FLOWING FULL
BASED ON MANNING'S EQUATION n=0.012

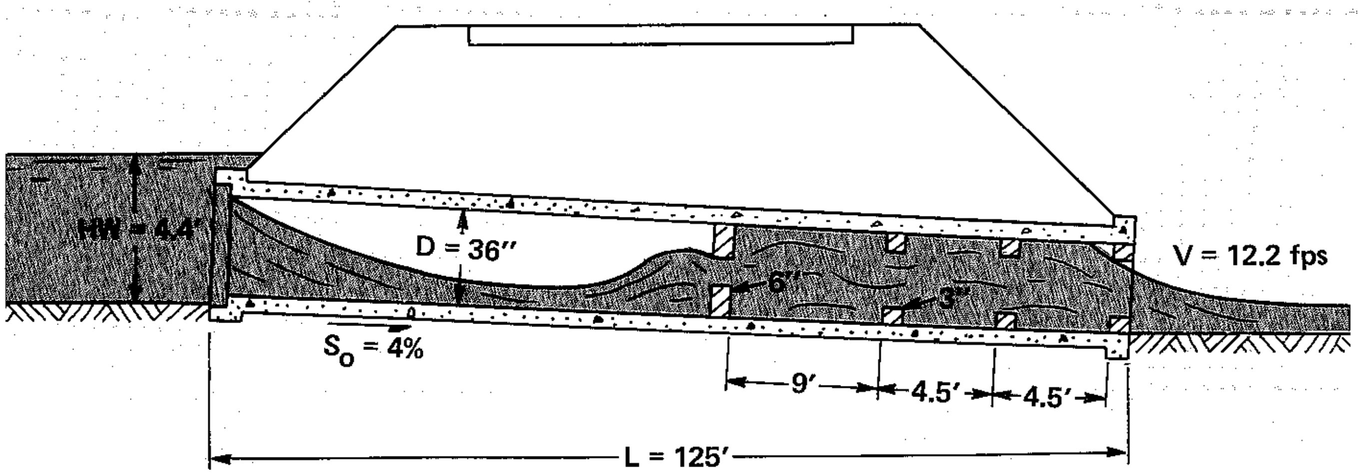


4. Determine Hydraulic cross-sectional area at last ring
   Pipe diameter = 36 inches
   Ring diameter = 30 inches
   From Table A-8 (on page 94) hydraulic cross-sectional area = 4.91 sq. ft.
5. Outlet Velocity
   $V = Q_d/A$
   $V = 60/4.91 = 12.2$ fps

Answer: Therefore, use three downstream elements, 3 inches high, spaced 4.5 feet, preceded by one upstream element, 6 inches high, spaced 9 feet as illustrated.

FIGURE 18 RELATIVE VELOCITY AND FLOW IN CIRCULAR PIPE FOR ANY DEPTH OF FLOW





FREE SURFACE FLOW DESIGN PROCEDURE

Based upon the free surface flow test results, the following design procedure is suggested:

1. Select required pipe size based on the hydraulic design procedures presented in Hydraulic Engineering Circular Nos. 5 or 10 prepared by the Federal Highway Administration or the Concrete Pipe Design Manual published by the American Concrete Pipe Association.
2. For culverts operating under inlet control, determine outlet velocity by means of Manning's Formula.
3. If velocity reduction is desired, select a pipe diameter within the following range:

$$\left[\frac{Q^2}{0.10g}\right]^{1/5} \leqq D \leqq \left[\frac{Q^2}{0.044g}\right]^{1/5}$$

   where Q = design discharge
   g = acceleration due to gravity (32.2)

   The five dissipator rings will be placed within this pipe diameter.

4. Select a roughness element size for the dissipator rings with a height-diameter ratio between 0.10 and 0.15,

$$0.10 \leqq K/D \leqq 0.15$$

   and a spacing-diameter ratio between 1.5 and 2.5.

$$1.5 \leqq L/D \leqq 2.5$$

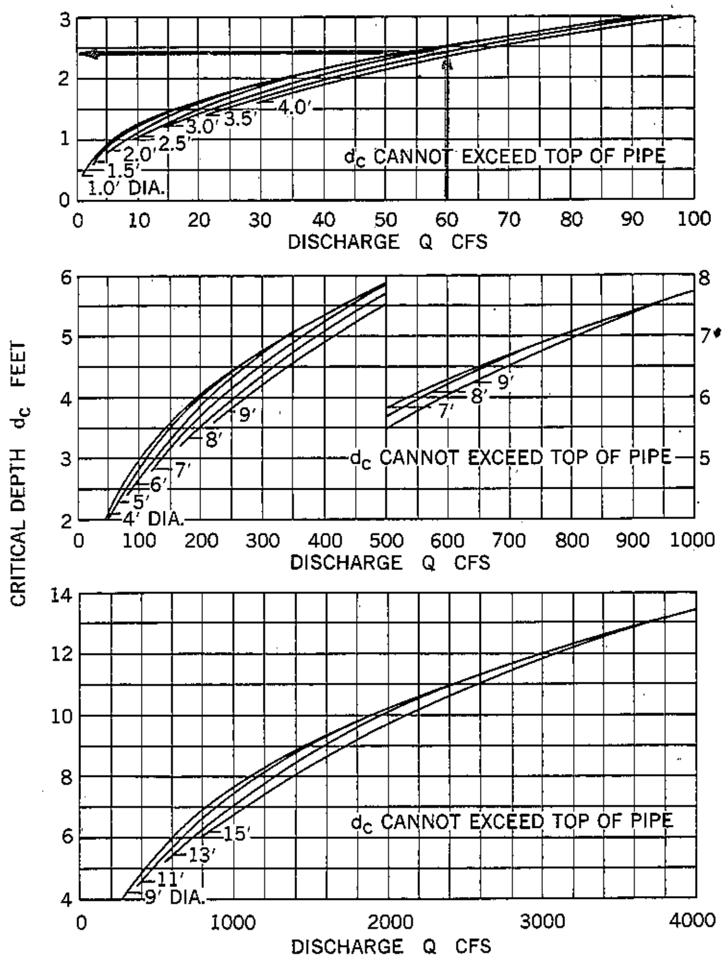
5. Determine hydraulic cross-sectional area at last dissipator ring based upon critical depth.
6. Divide the design discharge by the resultant area determined in Step 5 to determine the outlet velocity.

Example 2.

Given: Same as Example 1.

Find: Size and spacing of roughness elements for free surface flow conditions.

FIGURE 25 **CRITICAL DEPTH CIRCULAR PIPE**



Solution:

1. Check culvert control (see Example 1).
   Inlet control governs.
2. Determine outlet velocity (see Example 1).
   $Q_{full} = 145$ cfs
   $V_{full} = 20.5$ fps
   $Q_{design} = 60$ cfs
   $V_{design} = 19.3$ fps
3. Velocity reduction desired, select pipe diameter for culvert outlet.

$$\left[\frac{Q^2}{0.10g}\right]^{1/5} \leqq D \leqq \left[\frac{Q^2}{0.044g}\right]^{1/5}$$

$$\left[\frac{(60)^2}{(0.10)\ (32.2)}\right]^{1/5} \leqq D \leqq \left[\frac{(60)^2}{(0.044)\ (32.2)}\right]^{1/5}$$

$$4.0 \leqq D \leqq 4.8$$

   Try a 48-inch diameter pipe.

4. Select roughness element size and spacing.

   Size— $0.10 \leqq \frac{K}{D} \leqq 0.15$

$$4.8 \leqq K \leqq 6.0$$

   Try K = 5 inches.

Spacing— $1.5 \leq \frac{L}{D} \leq 2.5$

$72 \leq L \leq 120$

Spacing of five elements between 6 and 10 feet allows placing one element in each of five last culvert sections.

5. Determine hydraulic cross-sectional area of last ring

   inside pipe diameter = 48-inches

   inside ring diameter = 38-inches (3.2 feet)

   From Figure 25, page 202, Concrete Pipe Design Manual, for $D = 3.2$-feet, $d_c = 2.45$-feet

   $$\frac{d_c}{D} = \frac{2.45}{3.2} = 0.765$$

   From Table A-10, page 363, Concrete Design Manual,

   $$\frac{\text{Area}}{D^2} = 0.6446$$

   $\text{Area} = 0.6446\ (3.2)^2$

   $\text{Area} = 6.62$ sq. ft.

6. Determine outlet velocity.

   $V = Q/A$

   $V = 60/6.62$

   $V = 9.1$ fps

Answer: Therefore, use five elements, 5 inches high spaced 6 to 10 feet or the length of the pipe section if within this range, in the last five sections of pipe which are increased to 48 inches in diameter.

Discussion

The joining of the two sizes of pipe could be accomplished by telescoping or slipping the 36-inch pipe into the 48-inch pipe for at least the length of a normal joint and using normal sealing materials in the annular space.

Although the velocity reduction is somewhat greater for free surface flow than for full flow conditions, the method used should be selected only after a complete review of the economics, installation procedures and requirements of the project. Early consultation with the concrete pipe producer is suggested to take full advantage of manufacturing capabilities and design details.

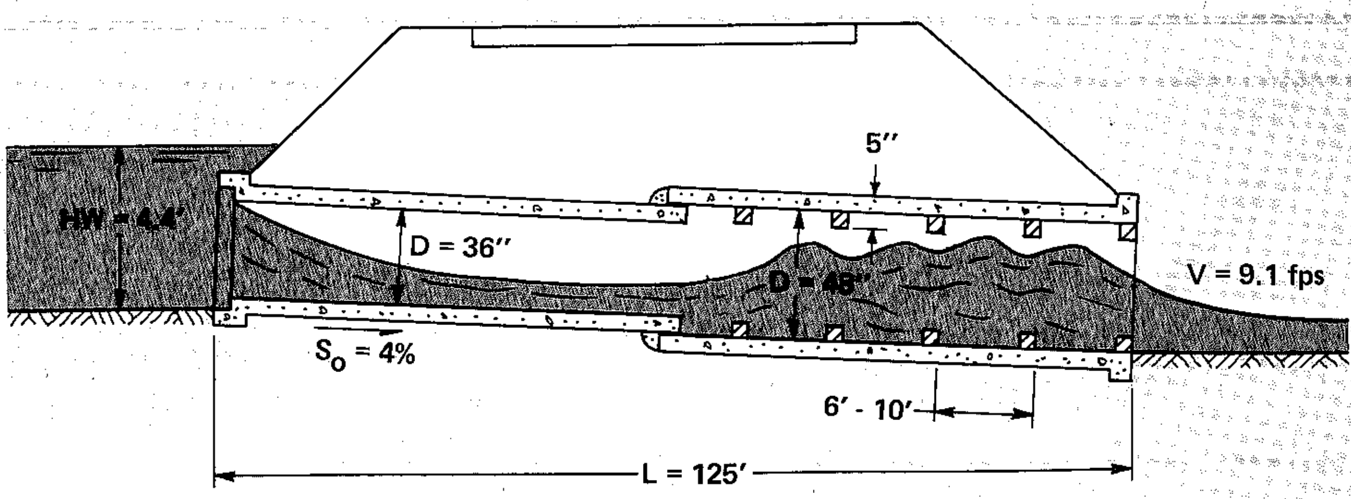


TABLE A-10

AREA, WETTED PERIMETER AND HYDRAULIC RADIUS OF PARTIALLY FILLED CIRCULAR PIPE

$\frac{d}{D}$	$\frac{\text{area}}{D^2}$	$\frac{\text{wet. per.}}{D}$	$\frac{\text{hyd. rad.}}{D}$	$\frac{d}{D}$	$\frac{\text{area}}{D^2}$	$\frac{\text{wet. per.}}{D}$	$\frac{\text{hyd. rad.}}{D}$
0.01	0.0013	0.2003	0.0066	0.51	0.4027	1.5908	0.2531
0.02	0.0037	0.2838	0.0132	0.52	0.4127	1.6108	0.2561
0.03	0.0069	0.3482	0.0197	0.53	0.4227	1.6308	0.2591
0.04	0.0105	0.4027	0.0262	0.54	0.4327	1.6509	0.2620
0.05	0.0147	0.4510	0.0326	0.55	0.4426	1.6710	0.2649
0.06	0.0192	0.4949	0.0389	0.56	0.4526	1.6911	0.2676
0.07	0.0242	0.5355	0.0451	0.57	0.4625	1.7113	0.2703
0.08	0.0294	0.5735	0.0513	0.58	0.4723	1.7315	0.2728
0.09	0.0350	0.6094	0.0574	0.59	0.4822	1.7518	0.2753
0.10	0.0409	0.6435	0.0635	0.60	0.4920	1.7722	0.2776
0.11	0.0470	0.6761	0.0695	0.61	0.5018	1.7926	0.2799
0.12	0.0534	0.7075	0.0754	0.62	0.5115	1.8132	0.2821
0.13	0.0600	0.7377	0.0813	0.63	0.5212	1.8338	0.2842
0.14	0.0668	0.7670	0.0871	0.64	0.5308	1.8546	0.2862
0.15	0.0739	0.7954	0.0929	0.65	0.5404	1.8755	0.2881
0.16	0.0811	0.8230	0.0986	0.66	0.5499	1.8965	0.2899
0.17	0.0885	0.8500	0.1042	0.67	0.5594	1.9177	0.2917
0.18	0.0961	0.8763	0.1097	0.68	0.5687	1.9391	0.2933
0.19	0.1039	0.9020	0.1152	0.69	0.5780	1.9606	0.2948
0.20	0.1118	0.9273	0.1206	0.70	0.5872	1.9823	0.2962
0.21	0.1199	0.9521	0.1259	0.71	0.5964	2.0042	0.2975
0.22	0.1281	0.9764	0.1312	0.72	0.6054	2.0264	0.2987
0.23	0.1365	1.0003	0.1364	0.73	0.6143	2.0488	0.2998
0.24	0.1449	1.0239	0.1416	0.74	0.6231	2.0714	0.3008
0.25	0.1535	1.0472	0.1466	0.75	0.6318	2.0944	0.3017
0.26	0.1623	1.0701	0.1516	0.76	0.6404	2.1176	0.3025
0.27	0.1711	1.0928	0.1566	0.77	0.6489	2.1412	0.3032
0.28	0.1800	1.1152	0.1614	0.78	0.6573	2.1652	0.3037
0.29	0.1890	1.1373	0.1662	0.79	0.6655	2.1895	0.3040
0.30	0.1982	1.1593	0.1709	0.80	0.6736	2.2143	0.3042
0.31	0.2074	1.1810	0.1755	0.81	0.6815	2.2395	0.3044
0.32	0.2167	1.2025	0.1801	0.82	0.6893	2.2653	0.3043
0.33	0.2260	1.2239	0.1848	0.83	0.6969	2.2916	0.3041
0.34	0.2355	1.2451	0.1891	0.84	0.7043	2.3186	0.3038
0.35	0.2450	1.2661	0.1935	0.85	0.7115	2.3462	0.3033
0.36	0.2546	1.2870	0.1978	0.86	0.7186	2.3746	0.3026
0.37	0.2642	1.3078	0.2020	0.87	0.7254	2.4038	0.3017
0.38	0.2739	1.3284	0.2061	0.88	0.7320	2.4341	0.3008
0.39	0.2836	1.3490	0.2102	0.89	0.7384	2.4655	0.2996
0.40	0.2934	1.3694	0.2142	0.90	0.7445	2.4981	0.2980
0.41	0.3032	1.3898	0.2181	0.91	0.7504	2.5322	0.2963
0.42	0.3130	1.4101	0.2220	0.92	0.7560	2.5681	0.2944
0.43	0.3229	1.4303	0.2257	0.93	0.7612	2.6061	0.2922
0.44	0.3328	1.4505	0.2294	0.94	0.7662	2.6467	0.2896
0.45	0.3428	1.4706	0.2331	0.95	0.7707	2.6906	0.2864
0.46	0.3527	1.4907	0.2366	0.96	0.7749	2.7389	0.2830
0.47	0.3627	1.5108	0.2400	0.97	0.7785	2.7934	0.2787
0.48	0.3727	1.5308	0.2434	0.98	0.7816	2.8578	0.2735
0.49	0.3827	1.5508	0.2467	0.99	0.7841	2.9412	0.2665
0.50	0.3927	1.5708	0.2500	1.00	0.7854	3.1416	0.2500

TABLE A-8

AREAS OF CIRCULAR SECTIONS (Square Feet)

Diameter: Inches	Diameter: Feet and inches	0	⅛	¼	⅜	½	⅝	¾	⅞
0	0-0		.0001	.0003	.0008	.0014	.0021	.0031	.0042
1	0-1	.0055	.0069	.0085	.0103	.0123	.0144	.0167	.0192
2	0-2	.0218	.0246	.0276	.0308	.0341	.0376	.0413	.0451
3	0-3	.0491	.0533	.0576	.0621	.0668	.0717	.0767	.0819
4	0-4	.0873	.0928	.0985	.1044	.1104	.1167	.1231	.1296
5	0-5	.1364	.1433	.1503	.1576	.1650	.1726	.1803	.1883
6	0-6	.1963	.2046	.2131	.2217	.2304	.2394	.2485	.2578
7	0-7	.2673	.2769	.2867	.2967	.3068	.3171	.3276	.3382
8	0-8	.3491	.3601	.3712	.3826	.3941	.4057	.4176	.4296
9	0-9	.4418	.4541	.4667	.4794	.4922	.5053	.5185	.5319
10	0-10	.5454	.5591	.5730	.5871	.6013	.6157	.6303	.6450
11	0-11	.6600	.6750	.6903	.7057	.7213	.7371	.7530	.7691
12	1-0	.7854	.8018	.8185	.8353	.8522	.8693	.8866	.9041
13	1-1	.9218	.9396	.9575	.9757	.9940	1.013	1.031	1.050
14	1-2	1.069	1.088	1.108	1.127	1.147	1.167	1.187	1.207
15	1-3	1.227	1.248	1.268	1.289	1.310	1.332	1.353	1.375
16	1-4	1.396	1.418	1.440	1.462	1.485	1.507	1.530	1.553
17	1-5	1.576	1.600	1.623	1.647	1.670	1.694	1.718	1.743
18	1-6	1.767	1.792	1.817	1.842	1.867	1.892	1.917	1.943
19	1-7	1.969	1.995	2.021	2.047	2.074	2.101	2.127	2.154
20	1-8	2.182	2.209	2.237	2.264	2.292	2.320	2.348	2.377
21	1-9	2.405	2.434	2.463	2.492	2.521	2.551	2.580	2.610
22	1-10	2.640	2.670	2.700	2.731	2.761	2.792	2.823	2.854
23	1-11	2.885	2.917	2.948	2.980	3.012	3.044	3.076	3.109
24	2-0	3.142	3.174	3.207	3.241	3.274	3.307	3.341	3.375
25	2-1	3.409	3.443	3.477	3.512	3.547	3.581	3.616	3.652
26	2-2	3.687	3.723	3.758	3.794	3.830	3.866	3.903	3.939
27	2-3	3.976	4.013	4.050	4.087	4.125	4.162	4.200	4.238
28	2-4	4.276	4.314	4.353	4.391	4.430	4.469	4.508	4.547
29	2-5	4.587	4.627	4.666	4.706	4.746	4.787	4.827	4.868
30	2-6	4.909	4.950	4.991	5.032	5.074	5.115	5.157	5.199
31	2-7	5.241	5.284	5.326	5.369	5.412	5.455	5.498	5.541
32	2-8	5.585	5.629	5.673	5.717	5.761	5.805	5.850	5.895
33	2-9	5.940	5.985	6.030	6.075	6.121	6.167	6.213	6.259
34	2-10	6.305	6.351	6.398	6.445	6.492	6.539	6.586	6.634
35	2-11	6.681	6.729	6.777	6.825	6.874	6.922	6.971	7.020
36	3-0	7.069	7.118	7.167	7.217	7.266	7.316	7.366	7.416
37	3-1	7.467	7.517	7.568	7.619	7.670	7.721	7.773	7.824
38	3-2	7.876	7.928	7.980	8.032	8.084	8.137	8.190	8.243
39	3-3	8.296	8.349	8.402	8.456	8.510	8.564	8.618	8.672
40	3-4	8.727	8.781	8.836	8.891	8.946	9.001	9.057	9.113
41	3-5	9.168	9.224	9.281	9.337	9.393	9.450	9.507	9.564
42	3-6	9.621	9.678	9.736	9.794	9.852	9.910	9.968	10.03
43	3-7	10.08	10.14	10.20	10.26	10.32	10.38	10.44	10.50
44	3-8	10.56	10.62	10.68	10.74	10.80	10.86	10.92	10.98
45	3-9	11.04	11.11	11.17	11.23	11.29	11.35	11.42	11.48
46	3-10	11.54	11.60	11.67	11.73	11.79	11.86	11.92	11.98
47	3-11	12.05	12.11	12.18	12.24	12.31	12.37	12.44	12.50
48	4-0	12.57	12.63	12.70	12.76	12.83	12.90	12.96	13.03
49	4-1	13.10	13.16	13.23	13.30	13.36	13.43	13.50	13.57

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